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Maximum Utilization of FM Subcarrier Bandwidth

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Abstract: In FM-FM telemetry and other frequency-division systems, maximum utilization of the FM subcarrier bandwidth is achieved if the generated spectrum width arising from modulation of the subcarrier is maintained equal to the receiving-equipment bandwidth. This equality must hold for all values of the data modulation frequency from zero to one-half the receiving-equipment bandwidth, if the available system performance is to be fully exploited. A technique for optimization of subcarrier information capacity (OSIC) ensures such operation. The magnitude of subcarrier frequency deviation is varied as a function of the input data frequency to achieve simultaneously optimum data resolution, signal-to-noise ratio, and frequency response.

Experimental studies have validated the design concept, and a theoretical comparison has been made between the performance of systems based upon present approaches and the OSIC constant-bandwidth approach.

In the opinion of the authors, a significant contribution of this study is the development of a long-needed "figure of merit" for evaluation of FM subcarrier system performance.

INTRODUCTION

Frequency-division techniques for multiplexing several data channels on a common wire or radio link have been widely used for many years. Frequency modulation-frequency modulation (FM-FM) radio telemetry continues to be the principal method for transmission of analog data, particularly for those applications requiring unsampled or continuous data. Along with the state-of-the-art advances in microminiaturization and the introduction of adaptive and digital techniques, there has been a gradual shift in emphasis from frequency-division systems to time-division systems. In particular, pulse-code modulation (PCM) for direct transmission of digital data has become common. However, since the principal users of telemetry (the test ranges) are now heavily instrumented with FM-FM telemetry equipment, it is expected that this FM-FM technique will continue to have application for at least another five years, and perhaps much longer for special applications requiring uninterrupted data transmission.

In present methods for using FM-FM telemetry, the telemetering engineer is required to adjust the frequency deviation of each subcarrier to a fixed value, determined by his estimation of the nature of the information to be transmitted. In some cases, the information may be stable, well-known, and predictable, but in most cases the opposite is true.

The Inter-Range Instrumentation Group (IRIG) standards for FM-FM telemetry provide for standardized subcarrier center frequencies, bandwidths, and frequency responses based on a modulation index (β) of five. These widely available standards (1,2,3) provide operational guidelines to the telemetering engineer. Proper use of these guidelines ordinarily ensures acceptable data readout if the input data frequency does not exceed the predicted value. Alternatively, if the telemetering engineer adjusts for a reduced frequency deviation to permit a higher data frequency response, he must accept a lower data signal-to-noise ratio and reduced data resolution. Hence, no adjustment provides optimum performance under other than very limited conditions.

NRL Problem R05-03; Project RF 008-04-41-4502. This is a final report on one phase of this problem; work is continuing on other phases. Manuscript submitted June 15, 1966.

A technique is described in this report which utilizes the full bandwidth available at each FM subcarrier and provides a system having maximum data resolution, signal-to-noise ratio, and frequency response possible. Such performance is accomplished by a technique that provides optimization of subcarrier information capacity (OSIC). In the OSIC approach, the frequency deviation of the subcarrier is varied as a function of the input data frequency to produce a generated spectrum which is nearly constant in width, having all the selected side currents within the receiver subcarrier bandwidth. This technique was first conceived by Stine* and investigated by Kluck.† Further investigation was delayed by pressure of other work having higher priority, so an invention disclosure§ and patent application¶ were not made until much later.

FUNDAMENTAL RELATIONSHIPS

The fundamentals of frequency modulation are presented in great detail by Hund (4), Tibbs (5), and Panter (6). Briefly, the process of FM results in a spectral energy distribution consisting of a carrier or center frequency (F_c) and a number of sidebands or side-current pairs offset in frequency on each side of the carrier. The frequency separations between the carrier and its nearest side currents and between any two adjacent side currents are always equal to the modulating frequency (F_m) if the modulating signal is a pure sine wave. Hence, we have side currents spaced at $\pm F_m$, $\pm 2F_m$, $\pm 3F_m$, *etc.*, relative to the carrier. The number of significant side-current pairs is determined by the deviation ratio or modulation index (β), which is defined as the ratio of the frequency deviation (ΔF) of the carrier (F_c) to the modulation frequency (F_m). Hence,

$$\beta = \Delta F / F_m. \tag{1}$$

The equation for the instantaneous amplitude (E) of a carrier wave modulated by a single sine wave is:

$$E = A \sin (2\pi F_c t - \beta \cos 2\pi F_m t) \tag{2}$$

where A is the amplitude of the unmodulated carrier. By appropriate mathematical manipulations, Eq. (2) can be written to give the instantaneous amplitudes of the modulated carrier and each side current, as follows:

$$\begin{aligned} E = A \{ & J_o(\beta) \sin 2\pi F_c t \\ & + J_1(\beta) [\sin 2\pi (F_c + F_m) t - \sin 2\pi (F_c - F_m) t] \\ & + J_2(\beta) [\sin 2\pi (F_c + 2F_m) t + \sin 2\pi (F_c - 2F_m) t] + \ldots \\ & + J_n(\beta) [\sin 2\pi (F_c + nF_m) t + (-1)^n \sin 2\pi (F_c - nF_m) t] \} \end{aligned} \tag{3}$$

where $J_n(\beta)$ is the Bessel function of the first kind, of order n, for the argument (β). Hence, the relative amplitude of each of the side currents for a given modulation index can be determined

*P.T. Stine, "Prescribed Preemphasis of Subcarrier Oscillators," Unpublished Notes, NRL Notebook 7113, pp. 89-96, July 1948.

†J. H. Kluck, "A Means of Improving the Frequency Response and Attaining Optimum Signal to Noise Ratio in FM/FM Telemetering by the Use of a Prescribed Emphasis at the Subcarrier Oscillator," NRL Technical Memorandum CRL-71, Sept. 1949.

§P.T. Stine, "Automatic Frequency Deviation Control System for Subcarrier Oscillators," Invention Disclosure Docket No. NRL 4041, Oct. 1962.

¶P.T. Stine, "Automatic Frequency Deviation Control System for Subcarrier Oscillators," Patent Application, Sept. 1963.

from tabulated values of the Bessel function. Both Hund (4) and Tibbs (5) provide such tabulations and both suggest that, in practice, only those side currents having amplitudes equal to or greater than 1 percent of the unmodulated carrier need be considered as significant. As will be evidenced later in this report, the level at which side-current amplitudes become significant is more properly determined by the acceptable amount of distortion resulting from the loss of such side currents. Also, it will be seen that low-level side currents play a larger proportionate part and are more important when the modulation index is low than for high modulation indices.

Spectral distributions and side-current amplitudes for various low values of modulation index are shown in Figs. 1, 2, and 3. The same type of information for somewhat higher values of β are shown in Figs. 4, 5, and 6. It should be noted that the spectral distributions for very low values of β are quite similar to those obtained for amplitude modulation, in that there is only one significant side-current pair. As the modulation index is increased to five ($\beta = 5$) and

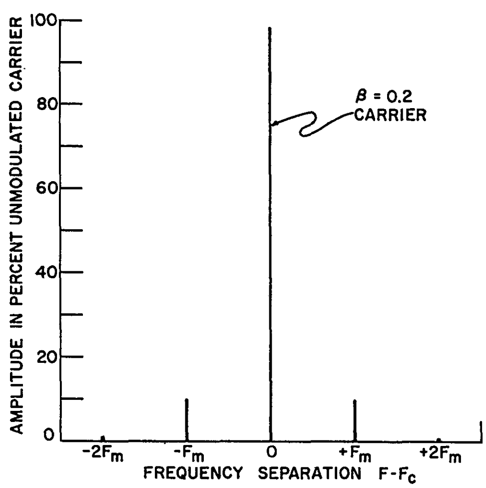

Fig. 1 – Spectral distribution for modulation index = 0.2

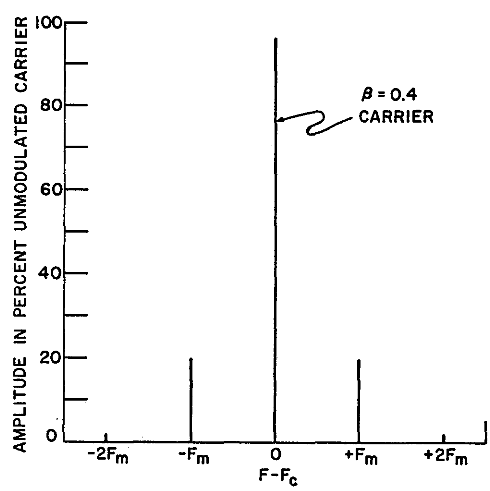

Fig. 2 – Spectral distribution for modulation index = 0.4

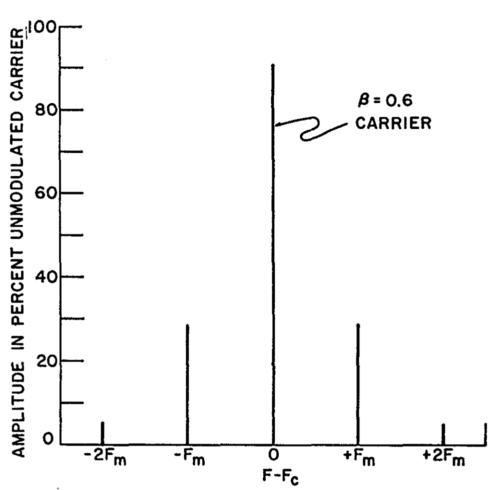

Fig. 3 – Spectral distribution for modulation index = 0.6

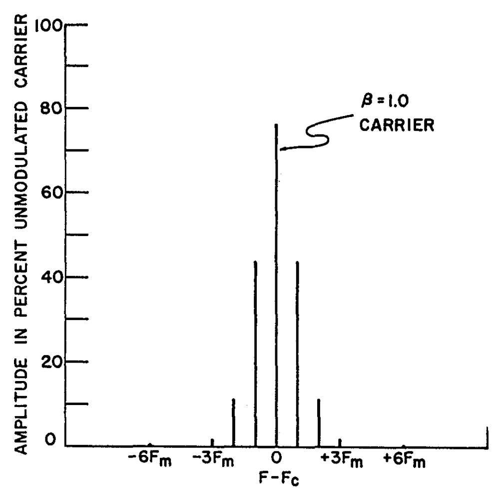

Fig. 4 – Spectral distribution for modulation index = 1.0

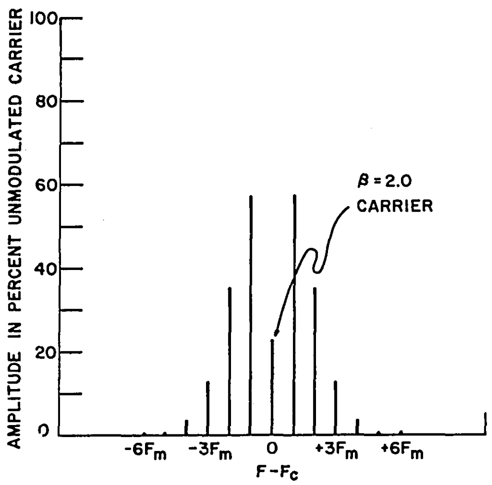

Fig. 5 – Spectral distribution for modulation index = 2.0

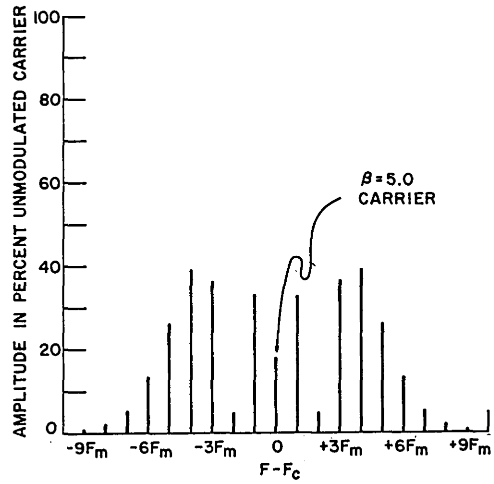

Fig. 6 – Spectral distribution for modulation index = 5.0

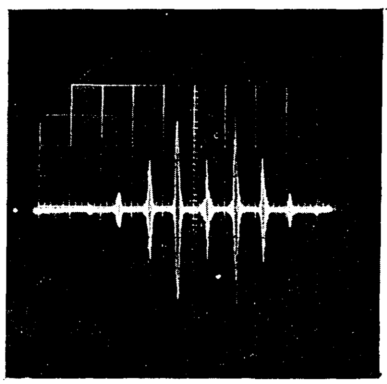

Fig. 7 – CRT presentation of spectrum for modulation index ≅ 2 (compare with Fig. 5)

higher, the modulation energy spreads into those side currents having wide separations from the carrier. Figure 7 is a photograph of an actual spectral distribution obtained with $\beta \cong 2$. Note the similarity with Fig. 5, which, of course, shows relative peak amplitudes instead of the peak-to-peak values obtained with a spectrum analyzer.

Consideration of Eq. (1) in relationship to Figs. 1 through 6 reveals that sizable side currents exist at frequency separations from the carrier which exceed the frequency deviation (ΔF). This fact is of great importance in that it indicates that the bandwidth required to pass the full spectrum width is considerably greater than twice the frequency deviation. From this standpoint, it is fortunate that actual passband shapes have skirts with finite slopes and do not resemble "idealized" passbands. The relationship of distortion to available bandwidth for various passband shapes and phase characteristics has been investigated mathematically by Sledge (7). Table 1 provides an indication of required bandwidth as a function of modulation index. For example, if side currents $\geqslant 1$ percent of the unmodulated carrier amplitude are considered significant, a bandwidth of 16 F_m is required for $\beta = 5$.

In FM-FM radio telemetry, a dual process of frequency modulations is used. The data of the information channels frequency modulate the subcarriers, which are then combined linearly and in turn frequency modulate the radio frequency carrier. At the receiving end, the rf carrier is demodulated by the FM receiver, the data-modulated subcarriers are selected

TABLE 1
Number of Side Current Pairs with Amplitude Exceeding Given Percentage of Unmodulated Carrier Amplitude

Modulation Index (β)	Number of Side Current Pairs			
	≥10%	≥5%	≥2%	≥1%
0.1	0	0	1	1
0.2	0	1	1	1
0.3	1	1	1	2
0.4	1	1	1	2
0.5	1	1	2	2
0.6	1	1	2	2
0.7	1	2	2	2
0.8	1	2	2	3
0.9	1	2	2	3
1.0	2	2	2	3
2.0	3	3	4	4
3.0	4	4	5	6
4.0	5	5	6	7
5.0	6	7	7	8
6.0	7	8	9	9
7.0	8	9	10	10
8.0	9	10	11	11
9.0	10	11	12	13
10.0	11	12	13	14
15.0	16	17	18	19
24.0	25	26	28	29

by bandpass filters, any incidental amplitude modulation is removed by a limiter, and the original data, with as little error and distortion as possible, is recreated as the demodulated output of the FM discriminators. The fundamental relationships reviewed previously apply equally to the frequency modulation of an rf carrier or lower frequency subcarriers. In the discussions to follow, however, we will be concerned only with the problems of frequency modulating subcarriers with data signals and demodulating them to recover the input data. The material presented may be applied, by and large, to any system using FM subcarriers, including hardwire telemetry, radio telemetry, telephone multiplex, and dc magnetic tape recorders.

DESIGN PHILOSOPHY

In all applications known to the authors, the modulation signal applied to the subcarrier is maintained at a constant preset amplitude selected by the user to give a fixed value of frequency deviation. As was shown in the section on fundamental relationships, maintenance at a constant value of frequency deviation does not provide a constant spectrum width. Instead, if ΔF is held constant, the spectrum width spreads and the receiving bandwidth required to

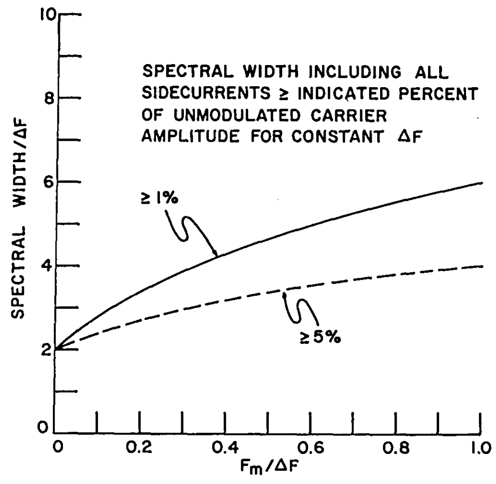

Fig. 8 — Spectral width as a function of modulation frequency for constant ΔF (spectral width includes all side currents $\geq$ indicated percent of unmodulated carrier amplitude)

pass all important side currents increases nonlinearly as a function of the modulating frequency. The functional relationship is approximately as shown in Fig. 8.

Standard practice, as recommended in the IRIG Telemetry Standards (1,2,3) is to adjust the frequency deviation to a full one-half bandwidth, *i.e.,* 7-1/2 or 15 percent of the subcarrier center frequency, depending upon the particular subcarrier band selected, and to limit the maximum modulation frequency to 20 percent of the maximum frequency deviation. Hence, the minimum modulation index recommended by the IRIG Telemetry Standards becomes $\beta = 5$. It is further indicated that modulation indices as low as $\beta = 1$ or less may be used. However, "low signal-to-noise ratios, possible increased harmonic distortion, and cross talk must be expected" (1,2,3). It is certainly true that one must expect lower signal-to-noise ratios at lower values of frequency deviation, but harmonic distortion and cross talk should not increase if the value of frequency deviation is properly selected to permit all significant side currents to be passed within the available equipment bandwidth (B).

The objective of the new technique described here is to achieve increased bandwidth utilization by providing constant spectrum width for all values of modulation frequency from zero to one-half of the available bandwidth (the maximum possible). The generated spectrum width is determined by the magnitude of the frequency deviation and is directly proportional to the amplitude of the modulating data signal. A constant spectrum width which includes side currents of selected amplitude can be achieved by the use of a proper filter characteristic between the modulation or data source and the subcarrier oscillator, which is frequency modulated as shown in Fig. 9. A second filter with exactly inverse characteristics of the first filter must be used following the demodulator at the receiving end in order that the amplitude of the data may be made to correspond to the correct value, *i.e.,* the amplitude at the source.

DESIGN IMPLEMENTATION

The original design concept* was based upon a plot of the allowable values of modulation index and frequency deviation to pass all side currents having amplitudes ≥ 1 percent of the unmodulated subcarrier amplitude for modulation frequencies up to 7.5 percent of the center frequency. The original plots, which have been redrawn in Fig. 10, were made using somewhat incomplete tables of Bessel functions to obtain side-current amplitudes. Figure 11 shows the allowable values of modulation index prepared from more complete tables of Bessel functions

*See the first two footnotes on page 2.

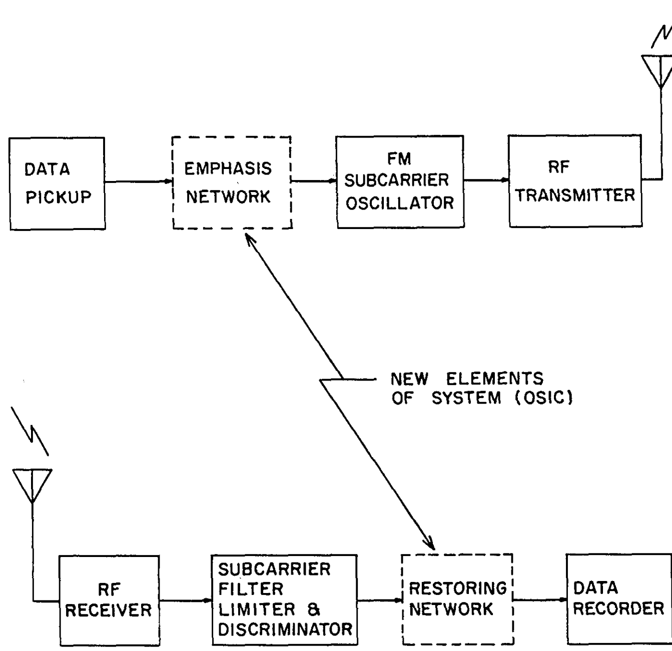

Fig. 9 – Application of OSIC to FM-FM telemetry (single channel shown)

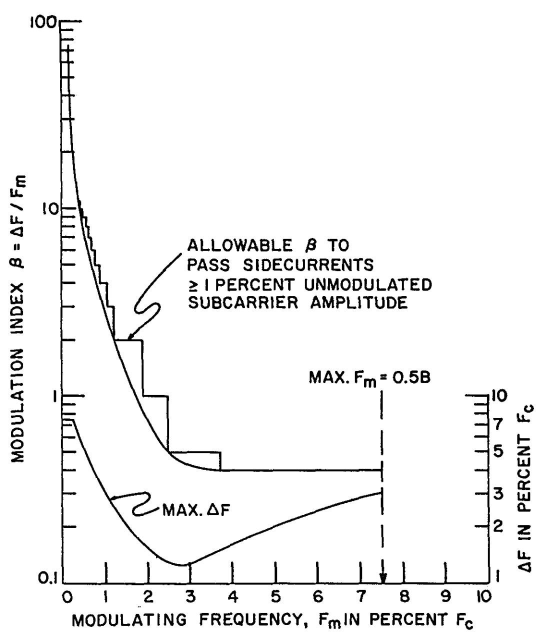

Fig. 10 – Modulation index β and frequency deviation ΔF *versus* modulating frequency F_m for bandwidth of ± 7.5 percent F_c

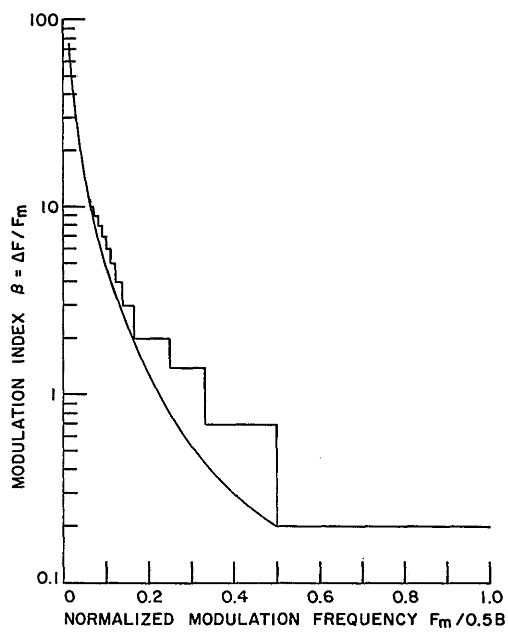

Fig. 11 – Modulation index β *versus* normalized modulation frequency to pass all side currents of amplitude ≥ 1 percent of unmodulated subcarrier amplitude

and plotted in a manner so as to be completely general in application. Note the "stairstep" characteristics caused by the discrete separation of side currents and the fact that a whole number of side-current pairs is passed by an "idealized" passband. The smooth curves have been drawn conservatively through or near the minimum β values. Figures 12, 13, and 14 are similar to Fig. 11, but assume different values for the "significant" side-current amplitudes. It is now very simple to take the product of the modulation indices shown and the normalized modulation frequency to obtain values of normalized frequency deviation. These data provide the desired OSIC filter-shape characteristic (Fig. 15). In Fig. 16, the same filter characteristics are plotted with attenuation shown in decibels.

The selection of the particular curve of Fig. 15 to be used as the OSIC filter requirement is a compromise between decreasing signal-to-noise ratio if lower values of side-current amplitudes are required to be passed and greater distortion resulting if they are not passed. Prior to actual design, construction, and test of OSIC networks, it was considered desirable to determine the compromise value of side-current amplitudes to be passed. This was accomplished by manually adjusting the amplitude of the modulating signal in accordance with the curves of Fig. 15 and measuring the resulting distortion. The frequency-modulated subcarrier signal was passed through a standard bandpass filter and hard limiter having the characteristics shown in Fig. 17, and demodulated by a standard subcarrier discriminator having an output filter flat from zero to the half bandwidth frequency (7.5 percent of F_c). For reference purposes, total-harmonic distortion was first measured at the subcarrier discriminator output with the bandpass filter removed and with ΔF constant at 7.5 percent F_c. The results are shown in Fig. 18. With the bandpass filter in the circuit, harmonic-distortion measurements were made for manual adjustments of modulation frequency amplitude corresponding to the 1, 5, and 10 percent curves of Fig. 15. The results are shown in Fig. 19, and the maximum values of distortion shown as a function of side-current amplitudes in Fig. 20. These data reveal that although loss of side currents with amplitudes of up to 5 percent of the unmodulated carrier amplitude may be regarded as not serious, significantly better results are obtained if all side currents with amplitudes of about 2 percent and higher are passed.

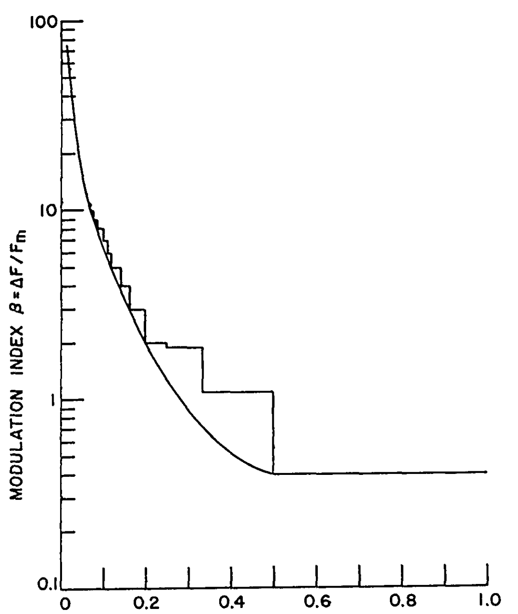

Fig. 12 – Modulation index β *versus* normalized modulation frequency to pass all side currents of amplitude $\geq$2 percent of unmodulated subcarrier amplitude

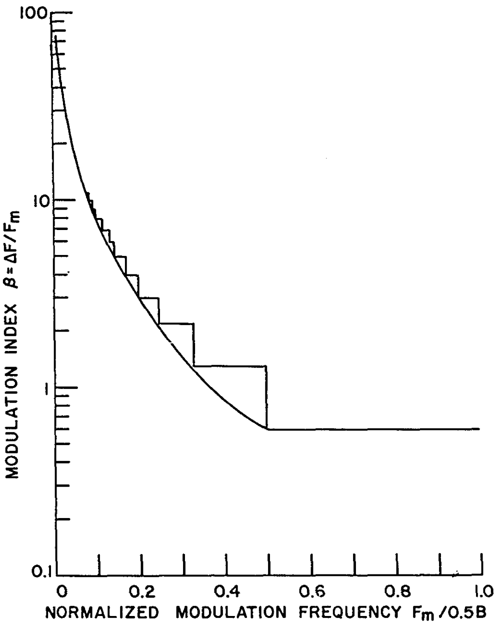

Fig. 13 — Modulation index β *versus* normalized modulation frequency to pass all side currents of amplitude $\geqslant 5$ percent of unmodulated subcarrier amplitude

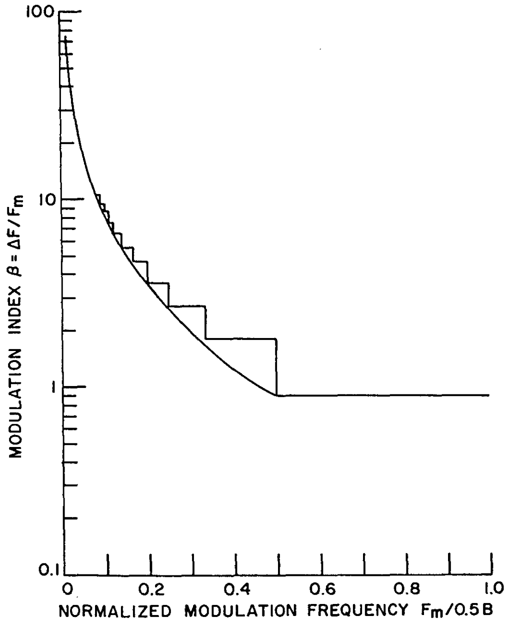

Fig. 14 — Modulation index β *versus* normalized modulation frequency to pass all side currents of amplitude $\geqslant 10$ percent of unmodulated subcarrier amplitude

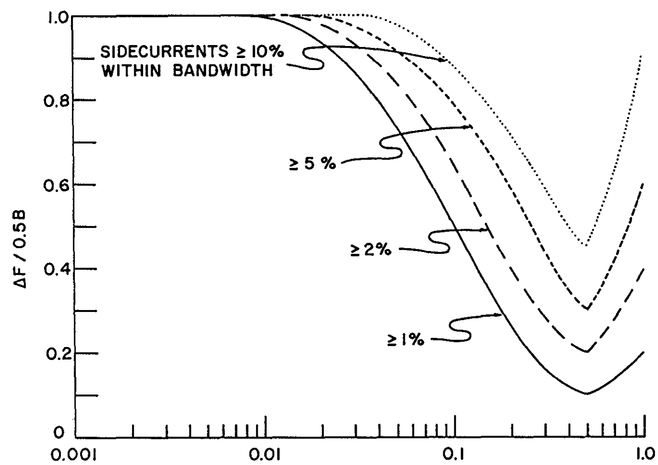

Fig. 15 — Desired OSIC filter-transfer characteristics for various values of significant side-current amplitudes

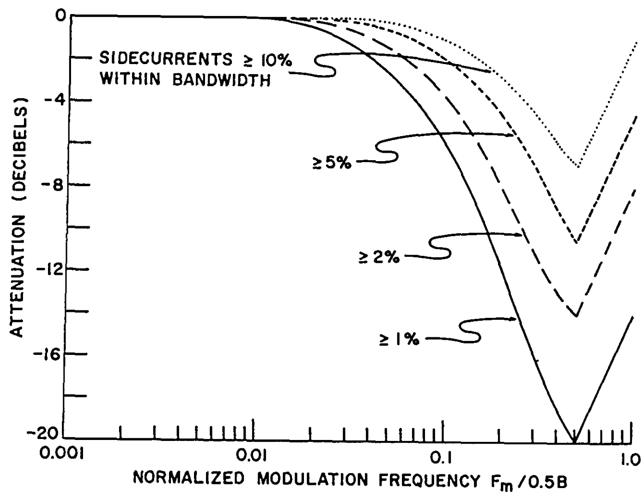

Fig. 16 – Desired OSIC filter-transfer characteristics in decibels

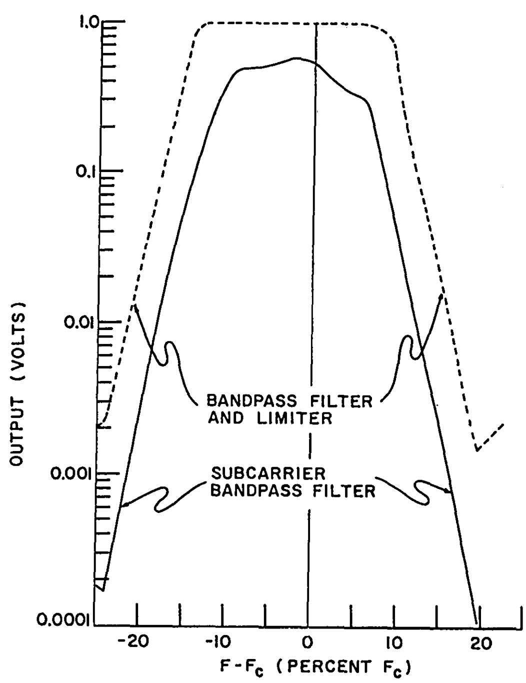

Fig. 17 – Subcarrier bandpass filter and limiter characteristics

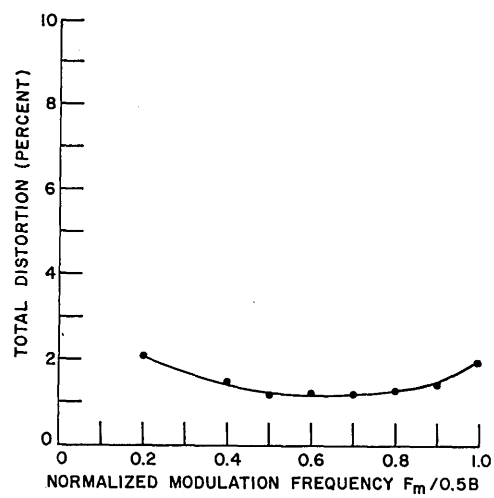

Fig. 18 – Total distortion attributable to modulation source and test set-up

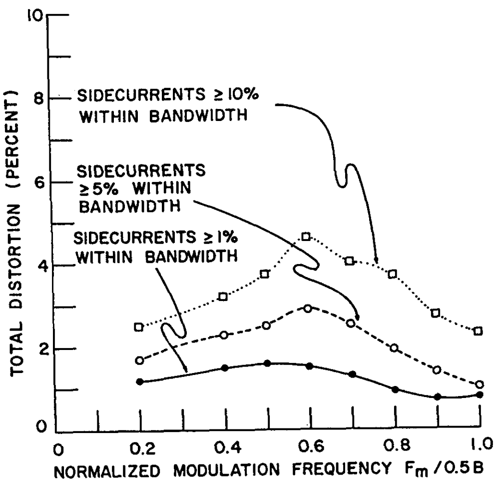

Fig. 19 – Relative significance of side-current amplitudes in terms of distortion resulting from their loss

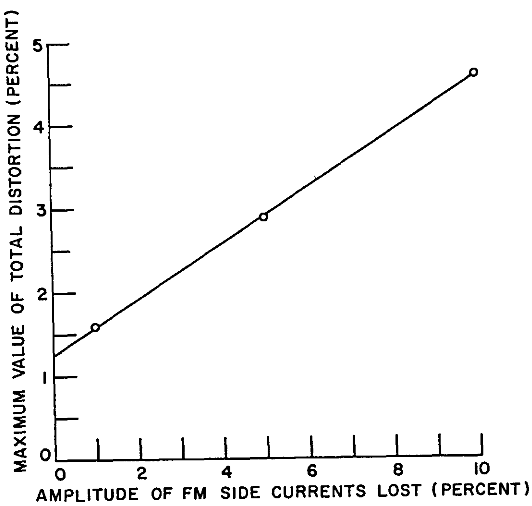

Fig. 20 – Measured distortion as a function of the amplitude of lost FM side currents

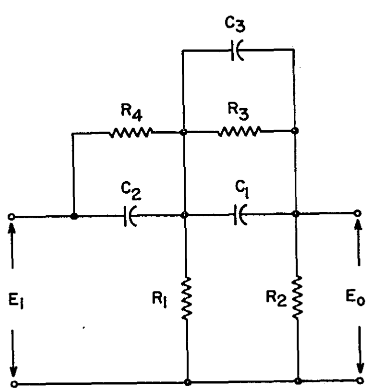

Fig. 21 – Seven-element RC network used as OSIC shaping circuit

Having chosen the acceptable degree of compromise relative to the selection of side-current amplitudes to be passed, we may now undertake the design, mechanization, and testing of an OSIC network to prove the validity of the design concept. Kluck* demonstrated the feasibility of using bridged RC parallel-T networks and inductance-capacitance networks but encountered some difficulty in obtaining accurate restoration of the data amplitude, primarily due to impedance-matching problems. From standpoints of size, weight, stability, cost, and simplicity, it is desirable to use a resistance-capacitance network mechanization for the desired shaping filter. Procedures developed by White (8) provide a method for designing a seven-element RC network suited to this application. Such a network is shown in Fig. 21. As a matter of convenience, it was decided to mechanize the test set-up on a standard 1300-cps telemetry subcarrier band. The network design procedure is given in Appendix A, and component design values are given in Table 2 for an OSIC shaping filter whose characteristics conservatively approximate the 2-percent side-current curve of Fig. 16. Calculated and measured values of the voltage-transfer characteristic of this filter are in close agreement (Fig. 22).

*NRL Technical Memorandum CRL-71 (see previous footnote).

TABLE 2
Basic Impedance Scaled and Frequency Scaled Component Values for 7-Element OSIC Network to Conservatively Pass All Side-Current Amplitudes ≥2% of Unmodulated Subcarrier Amplitude

Component	Basic Value at $\omega = 1$	z-Scaled 10^6 at $\omega = 1$	z-Scaled 10^6 and $\omega = 300$
R_1	0.141	141 K	1410 Ω
R_2	25.0	25 M	250 K
R_3	0.47505	475 K	4750 Ω
R_4	0.99701	997 K	9970 Ω
C_1	3.38983	3.39 μf	1.13 μf
C_2	1.41844	1.418 μf	0.473 μf
C_3	2.10716	2.107 μf	0.702 μf

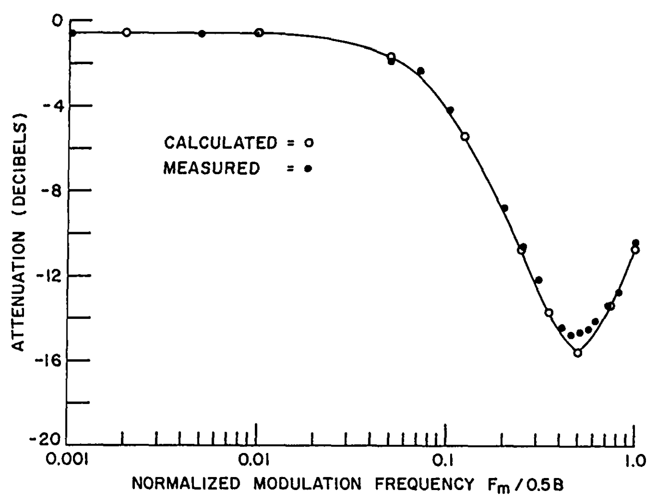

Fig. 22 – Seven-element OSIC network voltage-transfer characteristics

It will be recalled that a second filter having voltage-transfer characteristics exactly the inverse of the above filter is necessary to restore the output data to its original amplitude. Although it would be desirable to provide a passive filter with such characteristics, the authors know of no procedure for designing a reciprocal seven-element network. Hence, a feedback-amplifier approach using a second network identical to the one of Fig. 21 was used as the simplest and most accurate method of achieving an exactly inverse filter characteristic. Measured values of the feedback amplifier voltage-transfer characteristic are shown in Fig. 23. Comparison of Figs. 22 and 23 indicates that an inverse characteristic has been achieved within the accuracy of the measurement methods used. As a more accurate method for determining the exactness of amplitude restoration, network 1 was connected in series with the input terminal of the feedback amplifier with network 2 in its feedback path, and a comparison was made between the output and input voltages as a function of input frequency. The measurements shown in Fig. 24 indicate a restoration exactness of about ±1 percent. These results compare favorably with the accuracy of the measuring equipment.

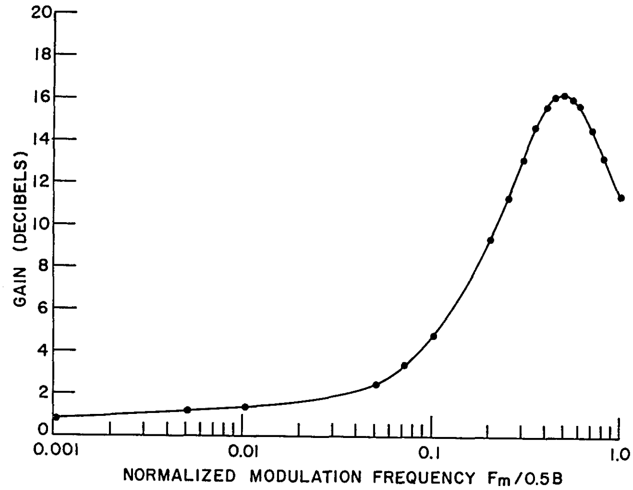

Fig. 23 — OSIC feedback amplifier restorer circuit measured voltage-transfer characteristics

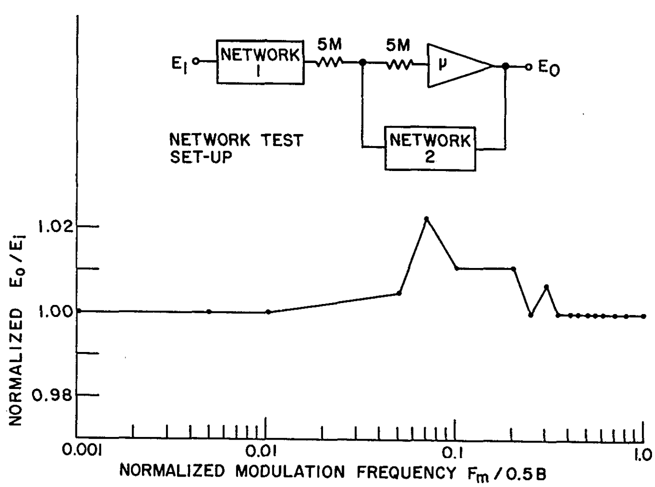

Fig. 24 — Performance of OSIC amplitude-restoring circuit

TEST RESULTS

In order to test the validity of the OSIC technique, equipment was set up as shown in Fig. 25 to simulate a single IRIG standard subcarrier band with $F_c = 1300$ cps and $B = \pm 7.5$ percent F_c. A hardwire link was used in lieu of the more common radio link. Switching from a constant-deviation configuration, as recommended by IRIG, to the OSIC constant-bandwidth configuration permitted ready comparison of performance measurements. The resistive voltage divider connected in the IRIG position of the switch simulated the dc attenuation of the OSIC network. Thus the output level of the function generator simulating the data source did not need readjustment upon switching to the other configuration. Figure 26 shows total distortion measured as a function of normalized modulating frequency for three constant-deviation settings which may be considered representative of (a) IRIG-recommended $\Delta F = 1.0$ (1/2 B), (b) modified IRIG with $\Delta F = 0.8$ (1/2 B), and (c) alternate IRIG with $\Delta F = 0.25$ (1/2 B). As previously

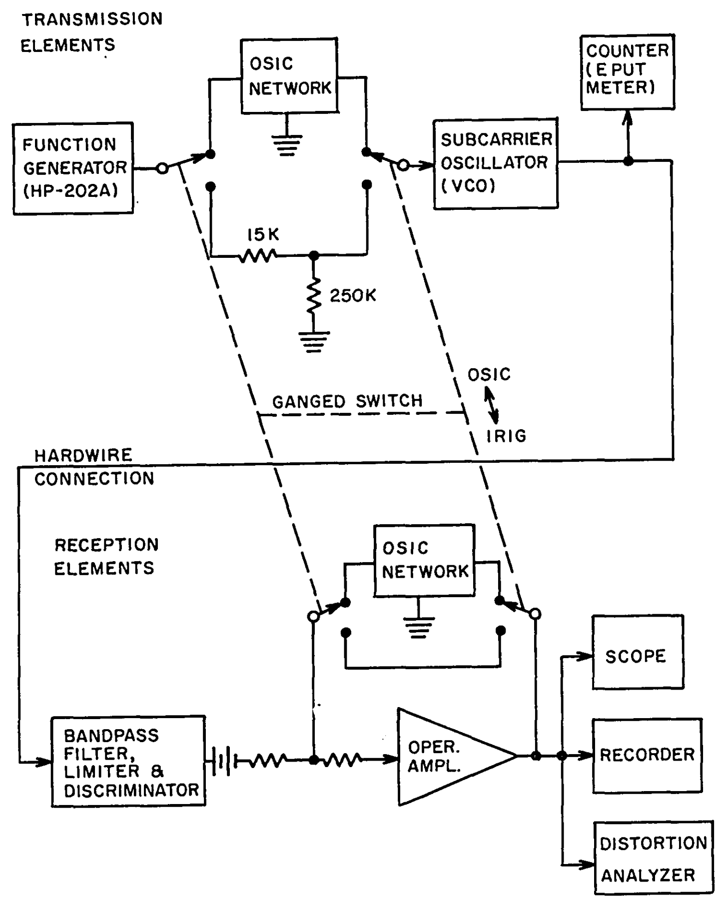

Fig. 25 – Block diagram of single-channel test and demonstration set-up

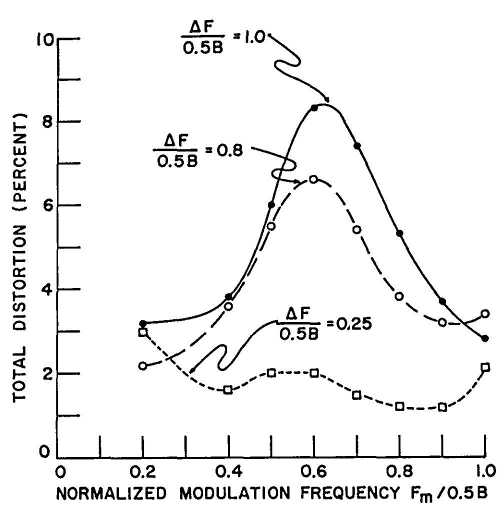

Fig. 26 – Total distortion measured using various constant-deviation adjustments

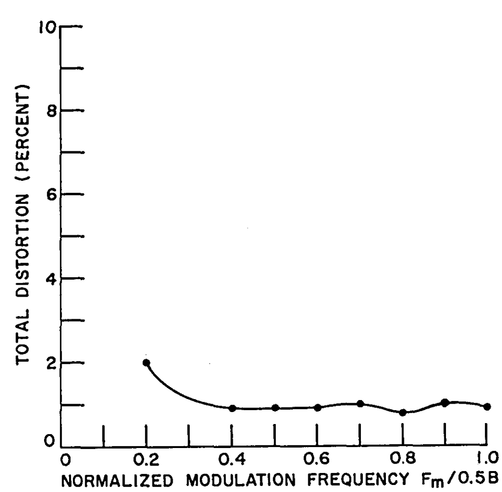

Fig. 27 – Total distortion using the OSIC constant-bandwidth configuration

indicated, Fig. 18 shows the distortion attributable to the function generator as a source when adjusted for constant full deviation of the subcarrier, and to other circuit elements. This distortion should not change with the mode of operation. Hence, except for the presence of the subcarrier bandpass filter, the data of the uppermost curve of Fig. 26 was taken with the same equipment set-up as used for taking the data of Fig. 18. Total distortion measurements made in the OSIC mode are shown in Fig. 27, and reveal a considerable improvement over those made in the IRIG mode.

PERFORMANCE EVALUATION

A figure of merit for comparative evaluation of various FM subcarrier systems has long been needed. Before a comparison of the OSIC constant-bandwidth approach with the various IRIG constant-deviation methods is undertaken, we digress to develop a suitable criterion.

It should be recognized that performance of a subcarrier channel is related to several factors, including signal-to-noise ratio (S/N), frequency response, and distortion. Accuracy and resolution (in terms of the number of incremental levels which can be distinguished) are largely determined by S/N ratio, which in turn is normally proportional to frequency deviation. If system noise predominates, the S/N ratio varies inversely as a function of ΔF. However, if data source noise (that which exists in the signal modulating the subcarrier) predominates, the S/N ratio will be essentially independent of the magnitude of ΔF. This is not usually the case in a well-designed system.

Determination of the effects of distortion upon performance is in most applications a subjective matter and therefore difficult to express mathematically. Subjectively, reduction of distortion below some low value (such as 0.001) does not affect performance significantly. Conversely, increasing distortion above some high value (such as 0.1) degrades performance rapidly. Based upon these considerations, the ratio of performance in the presence of distortion (P_d) to distortionless performance (P) is arbitrarily but conservatively defined as:

$$P_d/P = (1 - D/D_o) \tag{4}$$

wherein D is the measured distortion for a specific case and D_o is the value of distortion at which performance becomes completely unacceptable and hence may be considered as degraded to zero. This relationship is shown for three selected values of D_o in Fig. 28. As suggested previously, D_o would normally be chosen relative to the application.

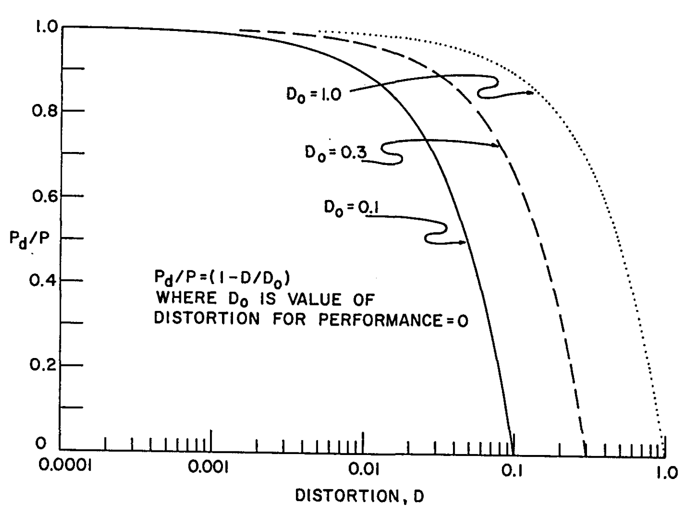

Fig. 28 – Normalized performance as a function of distortion

Let us now oversimplify somewhat and define a figure of merit, which we shall call relative performance (P_R) to be the product of normalized frequency deviation ($\Delta F/0.5\ B$), normalized frequency response ($F_m/0.5\ B$), and the normalized distortion effect ($1 - D/D_o$). Hence,

$$P_R = \left(\frac{\Delta F}{0.5\ B}\right)\left(\frac{F_m}{0.5\ B}\right)\left(1 - \frac{D}{D_o}\right). \tag{5}$$

The product of the first two terms of Eq. (5) will be recognized as the area under the IRIG constant-deviation constraints shown in Fig. 29. For the OSIC constant-bandwidth cases, the area under the curves must be found by a process of integration. The values for the areas under the various curves of Fig. 29, along with worst-case distortion values shown in Figs. 19, 26, and 27, are used to obtain values of relative performance as shown in Table 3. A value of 0.1 was chosen for D_o, since high values of distortion normally cannot be tolerated in telemetry and instrumentation applications.

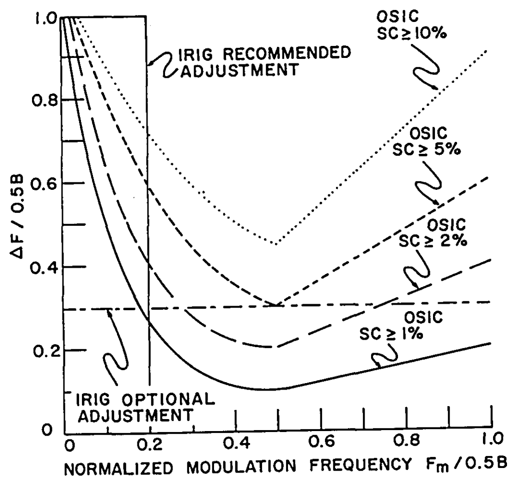

Fig. 29 — Performance of various constant-deviation (IRIG) and constant-bandwidth (OSIC) configurations

TABLE 3

Relative Performance of Various IRIG and OSIC Configurations

System Configuration	Area Under Curve (A)	Maximum Distortion (D)	$(1 - D/D_o)$	P_R
Recommended IRIG $\Delta F = 0.5B$	0.20	0.032	0.68	0.136
Modified IRIG $\Delta F = 0.4B$	0.16	≈0.022	0.78	0.125
Alternate IRIG $\Delta F = 0.15B$	0.30	0.029	0.71	0.213
OSIC Constant B $SC \geqslant 1\%$	0.20	0.016	0.84	0.168
OSIC Constant B $SC \geqslant 2\%$	0.37	≈0.020	0.80	0.296
OSIC Constant B $SC \geqslant 5\%$	0.52	0.029	0.71	0.369
OSIC Constant B $SC \geqslant 10\%$	0.68	0.046	0.54	0.367

It is interesting to note that Table 3 shows the relative performance of the OSIC case passing all side currents ⩾5 percent of the unmodulated carrier amplitude is numerically slightly greater than the sum of the relative performances of the recommended IRIG and alternate IRIG constant-deviation cases, which have been claimed to pass all side currents of the same amplitude (⩾5 percent). This fact tends to confirm that the OSIC technique combines the good features of the recommended IRIG and alternate IRIG methods.

CONCLUSIONS

The concept of optimization of subcarrier information capacity (OSIC) by the constant generated bandwidth techniques described herein has been shown to be capable of providing a substantial improvement in the performance of FM subcarrier bands over that achieved with presently used constant-deviation approaches. In fact, the use of OSIC techniques permits full bandwidth utilization at all times, thereby providing the maximum performance theoretically possible within the available bandwidth of the receiving equipment. It is further concluded that the validity of the design concepts have been demonstrated along with the feasibility, simplicity, and practicality of mechanization.

RECOMMENDATIONS

It is recommended that the concept of OSIC by constant-bandwidth techniques be applied in all future FM-FM telemetry systems. It is also recommended for application in other equipments or systems utilizing one or more FM subcarrier bands, such as dc magnetic tape recorders and certain multiplex telephone systems.

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Appendix A
DESIGN OF OSIC SHAPING NETWORK

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APPROXIMATING FUNCTION DETERMINATION

The required performance for the OSIC shaping filter is presumed to be specified by a plot of transmission as a function of frequency, as shown in Fig. A1. The frequency scale is normalized to $\omega = 1$ for the center frequency of the transmission "notch," and the minimum transmission level is designated by the numerical value z. At some fractional value $\omega = 1/n$ of notch center, the required transmission is specified by y. At zero frequency (dc), the desired transmission is indicated by the numerical value x. The frequency of minimum transmission and the three transmission levels of x, y, and z form a set of four performance constraints. The transfer function

$$\frac{E_{out}}{E_{in}} = \frac{s^2 + as + b}{s^2 + cs + d}, \tag{A1}$$

in which $s \equiv \sigma + j\omega$, contains the four parameters a, b, c, and d. For complex-conjugate zeros of transmission and negative-real poles, the function exhibits a magnitude-*versus*-frequency characteristic of the general form of Fig. A1. Regarding the four function parameters as adjustable variables, the immediate problem is that of meeting the previously stated four constraints.

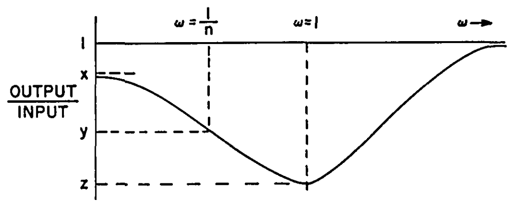

Fig. A1 – General form of network attenuation characteristic

The transfer function, magnitude M, is given by

$$M = \frac{s^2 + as + b}{s^2 + cs + d} = \left[\frac{\overline{b + s^2}^2 + (as/j)^2}{\overline{d + s^2}^2 + (cs/j)^2}\right]^{1/2} \tag{A2}$$

from which

$$M^2 = \frac{s^4 + (2b - a^2)\, s^2 + b^2}{s^4 + (2d - c^2)\, s^2 + d^2}. \tag{A3}$$

For steady-state, $s = j\omega$ ($s^2 = -\omega^2$, $s^4 = +\omega^4$), we have

$$M^2 = \frac{\omega^4 + (a^2 - 2b)\, \omega^2 + b^2}{\omega^4 + (c^2 - 2d)\, \omega^2 + d^2}. \tag{A4}$$

The minimum magnitude coincides with minimum M^2. Therefore, set $dM^2/d\omega = 0$ and get

$$[\omega^4 + (c^2 - 2d)\,\omega^2 + d^2]\,[4\omega^3 + 2(a^2 - 2b)\,\omega]$$
$$= [\omega^4 + (a^2 - 2b)\,\omega^2 + b^2]\,[4\omega^3 + 2(c^2 - 2d)\,\omega]. \tag{A5}$$

The combination of parameters required for a minimum transmission at $\omega = 1$ is thus,

$$(1 + c^2 - 2d + d^2)\,(2 + a^2 - 2b) = (1 + a^2 - 2b + b^2)\,(2 + c^2 - 2d)$$

or

$$\frac{c^2 + (1-d)^2}{c^2 + 2(1-d)} = \frac{a^2 + (1-b)^2}{a^2 + 2(1-b)}. \tag{A6}$$

Parenthetically, a moment's consideration of Eq. (A6) leads to the conclusion that neither b nor d can be exactly equal to unity for a minimum transmission at $\omega = 1$.

We are now in a position to write the following simultaneous equations. For $s = 0$, using Eq. (A1):

$$b = xd. \tag{A7}$$

For $\omega = 1$, using Eq. (A4):

$$\frac{a^2 + (1-b)^2}{c^2 + (1-d)^2} = z^2. \tag{A8}$$

For $\omega = 1/n$, using Eq. (A4):

$$\frac{n^2a^2 + (1 - n^2b)^2}{n^2c^2 + (1 - n^2d)^2} = y^2. \tag{A9}$$

Using Eqs. (A6) and (A8):

$$\frac{a^2 + 2\,(1-b)}{c^2 + 2\,(1-d)} = z^2. \tag{A10}$$

Substituting $b = xd$ from Eq. (A7) into Eqs. (A8), (A9), and (A10) yields

$$a^2 - z^2c^2 = z^2\,(1-d)^2 - (1-xd)^2 \tag{A11}$$

$$n^2\,(a^2 - y^2c^2) = y^2\,(1 - n^2d)^2 - (1 - n^2xd)^2 \tag{A12}$$

$$a^2 - z^2c^2 = 2\,[z^2\,(1-d) - (1-xd)]. \tag{A13}$$

Substitute a^2 from Eq. (A11) into Eq. (A13) to obtain

$$d = \left[\frac{1 - z^2}{x^2 - z^2}\right]^{1/2}. \tag{A14}$$

Substitute a^2 from Eq. (A11) into Eq. (A12) to obtain

$$c = \left[\frac{d^2\left([n^2-1]x^2 - n^2y^2 + z^2\right) + 2d\left(y^2 - z^2\right) - \frac{1}{n^2}\left([n^2-1] + y^2 - n^2z^2\right)}{y^2 - z^2}\right]^{1/2}. \tag{A15}$$

From Eq. (A11),

$$a = \{z^2\left[(1-d)^2 + c^2\right] - (1 - xd)^2\}^{1/2}. \tag{A16}$$

The transfer function parameters are determined by Eqs. (A14), (A15), (A7), and (A16) used together.

NUMERICAL EXAMPLE

Consider the data of Table A1 to be a specific requirement. Progressive substitution and use of the values determined into Eqs. (A14), (A15), (A7), and (A16) yields

$$d = 1.061 \tag{A17}$$

$$c = 6.043 \tag{A18}$$

$$b = 1.002 \tag{A19}$$

$$a = 1.003 \tag{A20}$$

TABLE A1
Transfer Function Data

Frequency (radians/sec)	Transmission (decibels)	Normalized Frequency	Transmission (numerical value)	Literal Symbol ($n = 4$)
0	−0.5	0	1/1.0593	x
38	−2.2	1/8		
75	−5.4	1/4	1/1.8629	y
150	−10.7	1/2		
300	−15.6	1	1/6.0256	z
600	−10.7	2		

NETWORK REALIZATION

For investigation of the possibility of network realization by the seven-element network of White (8), we substitute into Eq. (15) on page 4 of the referenced report:

$$\frac{a+c}{2} - \sqrt{\left(\frac{c-a}{2}\right)^2 - (d-b)}$$

$$< h \begin{cases} \text{either } ad/b \\ \text{or } \dfrac{a+c}{2} + \sqrt{\left(\dfrac{c-a}{2}\right)^2 - (d-b)} \end{cases}$$

and find the requirement for realizability

$$1.0148 < h < 1.0624. \tag{A21}$$

We may choose the network shown as Fig. 5a on page 5 of the referenced report. To make R_2 an exact number, arbitrarily select, within the required limits,

$$h = 1.043. \tag{A22}$$

Using the values specified by Eqs. (A17) through (A20) and (A22) in the expressions for the network component values, we find

$$R_1 = \frac{(h-a)\ (c-h) - (d-b)}{(h-a)\ (c-h)^2} = 0.141 \tag{A23}$$

$$R_2 = \frac{1}{h-a} = 25 \tag{A24}$$

$$R_3 = \frac{ad - bh}{ab\ (h-a)} = 0.47505 \tag{A25}$$

$$R_4 = \frac{1}{a} = 0.99701 \tag{A26}$$

$$c_1 = \frac{(c-h)\ (h-a)}{d-b} = 3.3898 \tag{A27}$$

$$c_2 = \frac{(h-a)\ (c-h)}{(h-a)\ (c-h) - (d-b)} = 1.4184 \tag{A28}$$

$$c_3 = \frac{a^2\ (h-a)}{ad - bh} = 2.1072 \tag{A29}$$

Impedance scaling from ohms to megohms and farads to microfarads is accomplished by multiplying all resistance values by 10^6 and capacitance values by 10^{-6}. With an eye to practical values, frequency scaling from the basic $\omega = 1$ design center frequency to the operating frequency of $\omega = 300$ may be accomplished by dividing all resistance values by 100 and all capacitance values by 3. These values are given in Table 2.

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13. ABSTRACT

In FM-FM telemetry and other frequency-division systems, maximum utilization of the FM subcarrier bandwidth is achieved if the generated spectrum width arising from modulation of the subcarrier is maintained equal to the receiving-equipment bandwidth. This equality must hold for all values of the data modulation frequency from zero to one-half the receiving-equipment bandwidth, if the available system performance is to be fully exploited. A technique for optimization of subcarrier information capacity (OSIC) ensures such operation. The magnitude of subcarrier frequency deviation is varied as a function of the input data frequency to achieve simultaneously optimum data resolution, signal-to-noise ratio, and frequency response.

Experimental studies have validated the design concept, and a theoretical comparison has been made between the performance of systems based upon present approaches and the OSIC constant-bandwidth approach.

In the opinion of the authors, a significant contribution of this study is the development of a long-needed "figure of merit" for evaluation of FM subcarrier system performance.

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